MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

ETM 4096 - DIGITAL SIGNAL PROCESSING

(BE/RE/TE)

14 OCTOBER 2019
2:30 P.M- 4:30 P.M.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 11 pages (including cover page) with 4 questions and 6-page list of formulas given in the appendix.

2. Attempt all **FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the answer Booklet provided.

QUESTION 1

a) Consider a discrete-time system

$$y[n] = x[n] + 3^n x[n-2]$$

i) State, with justifications, if this system is linear. [5 marks]

ii) State, with justifications, if this system is time-invariant. [4 marks]

b) A discrete-time linear time-invariant (LTI) system has impulse response:

$$h[n] = \{\underline{1}, -2, -1, 3\}$$

Determine the discrete-time Fourier transform (DTFT) of the system impulse response. [8 marks]

c) Figure Q1 shows a cascaded discrete-time system which relates input signal $x[n]$ and output signal $y[n]$. The impulse response of the subsystems are $h_a[n] = \{-2, \underline{5}\}$ and $h_b[n] = \{\underline{2}, 0, 3, -5\}$ respectively.

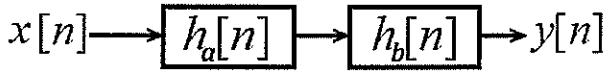


Figure Q1

i) Determine the effective system impulse response $h[n]$ of this cascaded system. [5 marks]

ii) State and justify if the effective system is stable and causal. [3 marks]

Continued..

QUESTION 2

a) A causal linear time-invariant (LTI) system is denoted by the system function:

$$H(z)=\frac{1.2z^{-1}}{1-0.4z^{-1}-0.32z^{-2}}$$

i) Determine all poles of $H(z)$ and state its region of convergence (ROCs). [4 marks]

ii) Determine the system impulse $h[n]$. [5 marks]

iii) Determine the values of $h[n]$ at n = -2, -1, 2, 3. [2 marks]

iv) Justify if the system is realizable. [2 marks]

b) A causal linear time-invariant (LTI) system is described by the following difference equation:

$$y[n]+1.44y[n-2]=x[n]-2x[n-1]-3x[n-2]$$

i) Determine system function $H(z)$. [4 marks]

ii) Plot the pole-zero plot of the system and label the region of convergence. [4 marks]

iii) Determine if the system is realizable. Justify your answer. [4 marks]

Continued...

QUESTION 3

a) The specifications of FIR lowpass filter given below :

Passband edge frequency	= 1.5 *kHz*	Stopband attenuation > 50*dB*
Stopband edge frequency	= 2.0 *kHz*	Sampling frequency = 8 *kHz*

i) Determine the order of the filter, N.

ii) Determine the cutoff frequency, ω_c.

iii) Determine the impulse response, $h[n]$.

iv) Compute only the middle five impulse response of the filter.

[2+2+3+5 marks]

b) The specifications of lowpass digital Butterworth filter is given below:

$$0.9 \leq |H(e^{j\omega})| \leq 1 \qquad |\omega| \leq 0.2\pi$$
$$|H(e^{j\omega})| \leq 0.2 \qquad 0.3\pi \leq |\omega| \leq \pi$$

Using the bilinear transformation method,

i) Determine the filter order, N.

ii) Determine the cutoff frequency, Ω_c.

iii) Determine the $H_a(s)$ and $H(z)$.

[5+2+6 Marks]

Continued...

QUESTION 4

a) Consider the finite-duration signal $x[n] = \{1, 2, 3, 1\}$, compute its four-point DFT by solving explicitly the 4-by-4 system of linear equations defined by the inverse DFT formula given below

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}nk}.$$

[10 marks]

b) The even samples of the DFT of a 9-point real signal $x[n]$ are given by

$X[0] = 3.1$

$X[2] = 2.5 + 4.6j$

$X[4] = -7 + 5.2j$

$X[6] = 9.3 + 6.3j$

$X[8] = 5.5 - 8.0j$

Determine the missing odd samples of the DFT. Use the properties of the DFT to solve this problem.

[10 marks]

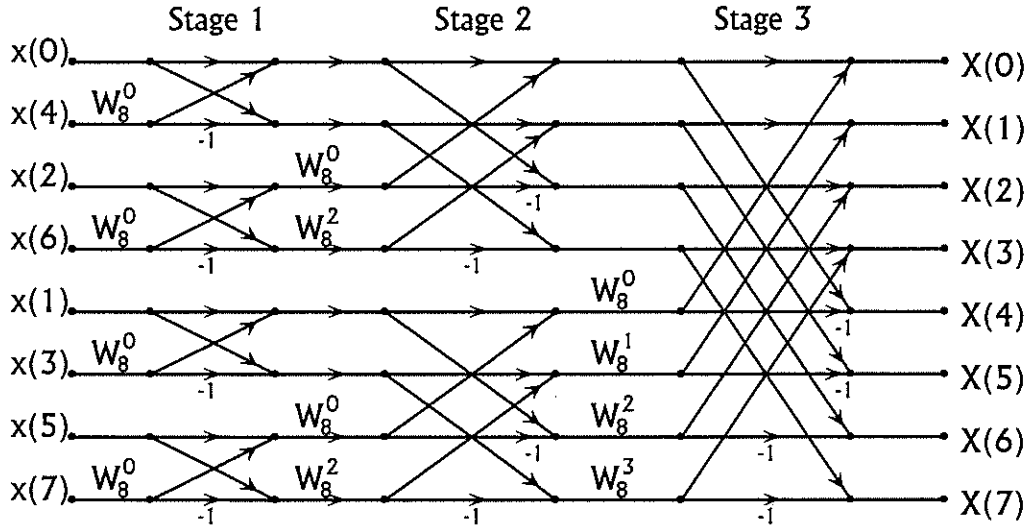


Figure Q4

c) Consider the eight-point decimation-in-time (DIT) flow graph in Figure Q4. Determine the gain of the "signal path" that goes from x(7) to X(2).

[5 Marks]

Continued...

APPENDIX

SUMMARY OF USEFUL TABLES

TABLE A1. DISCRETE-TIME FOURIER TRANSFORM PAIRS

Sequence	Discrete-Time Fourier Transform
$\delta[n]$	1
$\delta[n-n_o]$	$e^{-j\omega n_o}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega}+r^2e^{-j\omega})}$
$\frac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \omega_c < \lvert\omega\rvert < \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & otherwise \end{cases}$	$\frac{\sin[\omega(M+1)/2)]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{n=-\infty}^{\omega} 2\pi\delta(\omega-\omega_0+2\pi k)$
$\cos(\omega_0 n+\phi)$	$\sum_{n=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega-\omega_0+2\pi k)+\pi e^{-j\phi}\delta(\omega+\omega_0+2\pi k)]$

TABLE A2. SOME COMMON z-TRANSFORM PAIRS

The z-transform

Properties of the z-transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n]+by[n]$	$aX(z)+bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n-m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n]*y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common z-transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n-n_0]$	z^{-n_0}	Possibly $\forall z$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$

TABLE A3. SOME COMMON WINDOWS FOR FIR FILTER DESIGN

Ideal bandpass, n=0, 1, 2 …., N (Filter order given by N)

$$h[n]=\frac{w_2}{\pi}\sin c\left(\frac{w_2\left(n-\frac{N}{2}\right)}{\pi}\right)-\frac{w_1}{\pi}\sin c\left(\frac{w_1\left(n-\frac{N}{2}\right)}{\pi}\right)$$

Rectangular	$w[n]=\begin{cases}1 & 0\le n\le N\\ 0 & else\end{cases}$
Hanning	$w[n]=\begin{cases}0.5-0.5\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Hamming	$w[n]=\begin{cases}0.54-0.46\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Blackman	$w[n]=\begin{cases}0.42-0.5\cos\left(\frac{2n\pi}{N}\right)+0.08\cos\left(\frac{4n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Kaiser	$w[n]=\frac{I_0\left(\beta\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0(\beta)}\quad 0\le n\le N\quad \alpha=N/2$ where $\beta=\begin{cases}0.1102(A-8.7), & A>50\\ 0.5842(A-21)^{0.4}+0.07886(A-21), & 21\le A\le 50\\ 0, & A<21\end{cases}$ $N=\begin{cases}(A-7.95)/(2.285\Delta\omega), & A\ge 21\\ 5.655/\Delta\omega, & A<21\end{cases}$

TABLE A4. COMPARISON OF COMMONLY USED WINDOWS

Type of Window (Filter length = N+1)	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Stopband Attenuation, $20\log_{10}\delta$ (*dB*)	Transition Width ($\Delta\omega$ $\lvert w_p - w_s \rvert$)
Rectangular	-13	$4\pi/(N+1)$	-21	$1.8\pi/N$
Bartlett	-25	$8\pi/N$	-25	
Hanning	-31	$8\pi/N$	-44	$6.2\pi/N$
Hamming	-41	$8\pi/N$	-53	$6.6\pi/N$
Blackman	-57	$12\pi/N$	-74	$11\pi/N$

TABLE A5. IIR FILTER DESIGN

TABLE A5.1. THE COEFFICIENTS IN THE SYSTEM FUNCTION OF A NORMALIZED BUTTERWORTH LOW PASS FILTER (Ω_c=1) FOR ORDERS $1 \le N \le 8$

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.0000							
2	1.4142	1.0000						
3	2.0000	2.0000	1.0000					
4	2.6131	3.4142	2.6131	1.0000				
5	3.2361	5.2361	5.2361	3.2361	1.0000			
6	3.8637	7.4641	9.1416	7.4641	3.8637	1.0000		
7	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1.0000	
8	5.1258	13.1371	21.8462	25.6884	21.8462	13.1372	5.1258	1.0000

TABLE A5.2. CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \qquad |a| < 1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

TABLE A6. PARAMETERS OF ANALOG FILTER PROTOTYPES

Discrimination factor $d = \left[\dfrac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}\right]^{1/2}$

Selectivity factor $k = \dfrac{\Omega_p}{\Omega_s}$

Butterworth	Order, $N \geq \dfrac{\log d}{\log k}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$ where $s_k = \Omega_c e^{j(2k+1+N)\pi/(2N)}$
Chebyshev-I	Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = H_a(j0) \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$ where $s_k = -\Omega_p \sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j\Omega_p \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]$ $\epsilon = \sqrt{(1-\delta_p)^{-2}-1}$ $H_a(j0) = \begin{cases} 1, & \text{odd } N \\ (1+\epsilon^2)^{-0.5}, & \text{even } N \end{cases}$
Chebyshev-II	Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{s_k}{s'_k} \left(\dfrac{s - s'_k}{s - s_k}\right)$ where $s_k = \Omega_s\{-\sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]\}^{-1}$ $s'_k = j\Omega_s / \cos((2k+1)\pi/(2N))$ $\epsilon = 1/\sqrt{\delta_s^{-2}-1}$

Elliptic	$N \geq \dfrac{\log(16/d^2)}{\log(1/q)}$, $q = q_o + 2q_o^5 + 15q_o^9 + 150q_o^{13}$, $q_o = \dfrac{1}{2}\left[\dfrac{1-(1-k^2)^{1/4}}{1+(1-k^2)^{1/4}}\right]$

TABLE A7. DISCRETE-TIME FOURIER ANALYSIS

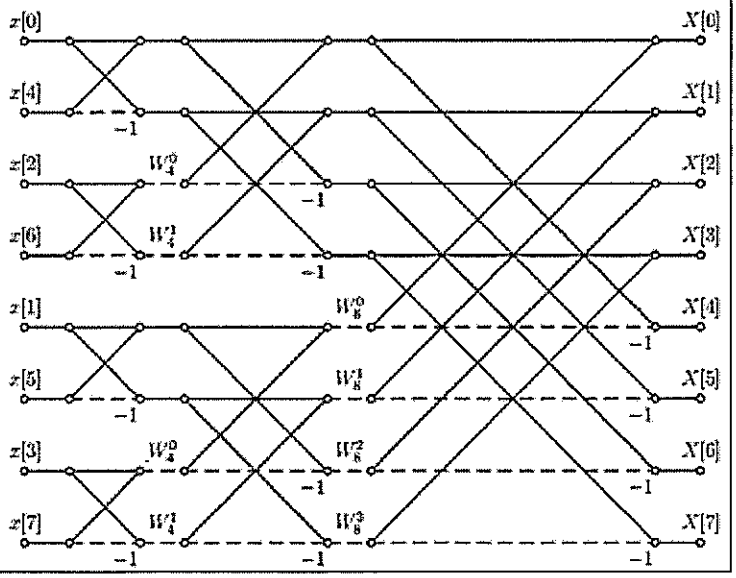
The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

Properties of the DFT

Property	$x[n]$	$X[k]$
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1X_1[k] + A_2X_2[k]$
Time shifting	$x[\langle n - n_0\rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k - k_0\rangle_N]$
Time reversal	$x[\langle -n\rangle_N]$	$X[\langle -k\rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k\rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \circledast Y[k]$

The decimation-in-time FFT



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